Exercise 1. Monetary policy evaluation and dynamic inconsistency problem

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1 Monetary policy evaluation

1. Suppose that there are three alternative options for monetary policy that lead to the following combinations of inflation and the unemployment rate (in per cent):

	Policy A	Policy B	Policy C
Unemployment rate	4	8	9
Inflation	4	3	2

Knowing that the loss function of the policymaker is given by:

$$Loss = unemployment + \theta * (Inflation - 2\%)^2$$

(a) Which is the best policy option assuming that $\theta = 1$?
Answer:

$$\text{PolicyA} : 4 + (4 - 2)^2 = 8 \quad (1)$$
$$\text{PolicyB} : 8 + (3 - 2)^2 = 9 \quad (2)$$
$$\text{PolicyC} : 9 + (2 - 2)^2 = 9 \quad (3)$$

The best policy is policy A.
(b) Suppose that preferences regarding inflation change to $\theta = 3$. Which policy is now the best one?
Answer:

$$\text{PolicyA} : 4 + 3(4 - 2)^2 = 16 \quad (4)$$
$$\text{PolicyB} : 8 + 3(3 - 2)^2 = 11 \quad (5)$$
$$\text{PolicyC} : 9 + 3(2 - 2)^2 = 9 \quad (6)$$

The best policy is policy C.
2. Suppose that you have to make decisions on two alternative options for monetary policy, A and B. Each policy produces uncertain results on the economy. Given your economic indicators and models, it is possible to make probabilistic statements regarding the impact of each policy on consumption. These are summarized in the table below.

	Policy A		Policy B	
Consumption	95	120	90	115
Probability	0.3	0.7	0.1	0.9

Knowing that utility is given by the following log-utility function:

$$Utility = ln(Consumption)$$

Compute the best policy assuming that the policymaker is trying to maximize expected utility. Comment your result.

Answer:

$$PolicyA : E(U) = 0.3 * ln(95) + 0.7 * ln(120) = 4.717$$
$$PolicyB : E(U) = 0.1 * ln(90) + 0.9 * ln(115) = 4.720$$

Policy B is the best one.

2 Dynamic inconsistency problem

Kydland and Prescott (1977) and Barro and Gordon (1983) provide a famous example of time inconsistency of inflation policy that gives support to the independence of central banks and their mandate of price stability. This exercise is derived from Mankiw, Macroeconomics, chapter 14.

We start with the Phillips curve, which describes the relation between unemployment (u) and inflation (π). In its modern version, the Phillips curve implies that inflation depends on expected inflation (π^e), the deviation of unemployment from its natural rate ($u - u^n$) and supply shocks (v):

$$\pi = \pi^e - \beta(u - u^n) + v \tag{7}$$

and $\beta(> 0)$ is a coefficient measuring the sensitivity of inflation to cyclical unemployment. For the purpose of this exercise we are going to assume that the supply shocks are zero ($v = 0$).

This curve can be solved in terms of unemployment:

$$u = u^n - \frac{1}{\beta}(\pi - \pi^e) \tag{8}$$

Now suppose that the policymaker can control inflation perfectly. We already know that the policymaker will need an instrument for this but as a simplification we will assume that the policymaker can perfectly choose the level of inflation. In addition, suppose that the policymaker is trying to minimize the following loss function:

$$Loss = u + \pi^2 \tag{9}$$

Please answer the following questions:

1 Assume that the policymaker decides its policy following a rule that is credible. This means that the public expectations for inflation policy coincide with the actual level of inflation chosen by the policymaker. If this is so, what is the level of inflation that the policy maker will choose and what is the value of the loss function?

Answer:

Rule based assessment - commitment

$$\pi = \pi^e - \beta(u - u^n)$$
$$PC : u = u^n - \frac{1}{\beta}(\pi - \pi^e)$$

Commitment implies

$$\pi^e = \pi \Rightarrow u = u^n$$
$$Loss = u + \pi^2$$
$$\frac{dLoss}{d\pi} = 0 \Leftrightarrow 2\pi = 0 \Leftrightarrow \pi = 0$$
$$Loss = u^n$$

2. Now assume that the policymaker decides by discretion. In this setting the policymaker will choose the level of inflation after the private agents have formed their expectations. However, when forming their expectations the agents take into account the subsequent optimal decision of the policymaker. Compute the resulting level of inflation and the value of the loss function.

Answer:

Under discretion:

1. Agents form their expectations
2. Central bank decides on optimal inflation rate without pre-commitment

Assume the expected inflation rate is π^e. Under discretion the central bank will minimize the loss function taking π^e as given. The economy functioning is given by the Phillips curve: ($u = u^n - \frac{1}{\beta}(\pi - \pi^e)$):

$$Loss = u + \pi^2 = (u^n - \frac{1}{\beta}(\pi - \pi^e)) + \pi^2$$
$$\frac{dLoss}{d\pi} = 0 \Leftrightarrow -\frac{1}{\beta} + 2\pi = 0$$
$$\pi = \frac{1}{2\beta}$$

Noting that the agents will anticipate the optimal decision of the central bank (setting $\pi^e = \pi$), the best is for the central bank to set $\pi = \pi^e$ implying:

$$u = u^n$$

The loss function will be:

$$Loss = u^n + (\frac{1}{2\beta})^2$$

3. Please repeat the previous questions assuming that the loss function is:

$$Loss = u + \lambda\pi^2 \tag{10}$$

where λ represents a stronger weight given to inflation ($\lambda > 1$)

Answer:

- Commitment: $\pi = 0; Loss = u^n$
- Discretion: $\pi = \frac{1}{2\lambda\beta}; Loss = u^n + \lambda(\frac{1}{2\lambda\beta})^2$

Note that given that ($\lambda > 1$), inflation will be lower under discretion than in the case when you have a weight of 1 in the loss function.

4. Repeat the previous questions assuming a quadratic loss function for both inflation and unemployment ($Loss = u^2 + \pi^2$).

Answer:

Rule based assessment - commitment.

From the Phillips curve we have:

$$\pi = \pi^e - \beta(u - u^n)$$
$$u = u^n - \frac{1}{\beta}(\pi - \pi^e)$$

Commitment implies that the central bank will credibly deliver on its promises. As a result, the expected inflation rate will coincide with actual inflation and so, from the workings of the Phillips curve, unemployment will be at the natural rate:

$$\pi^e = \pi \Rightarrow u = u^n$$

The central bank will choose the inflation rate to minimise the loss function (it is straightforward to see that the optimal inflation rate is zero):

$$Loss = u^2 + \pi^2$$

$$\frac{dLoss}{d\pi} = 0 \Leftrightarrow 2\pi = 0 \Leftrightarrow \pi = 0$$

The loss function will then be:

$$Loss = (u^n)^2$$

Discretion

Discretion implies that the policymaker will choose the level of inflation after the private agents have formed their expectations. However, when forming their expectations the agents take into account the subsequent optimal decision of the policymaker:

1. Agents form their expectations
2. Central bank decides on optimal inflation rate without pre-commitment

Let us assume the expected inflation rate is π^e. Under discretion the central bank will minimize the loss function taking π^e as given and the functioning of the economy given by the Phillips curve: $(u = u^n - \frac{1}{\beta}(\pi - \pi^e))$:

$$Loss = u^2 + \pi^2 = (u^n - \frac{1}{\beta}(\pi - \pi^e))^2 + \pi^2$$
$$\frac{dLoss}{d\pi} = 0 \Leftrightarrow -2(u^n - \frac{1}{\beta}(\pi - \pi^e))\frac{1}{\beta} + 2\pi = 0$$

Noting that the agents will anticipate the optimal decision of the central bank (setting $\pi^e = \pi$):

$$-\frac{u^n}{\beta} + \pi = 0$$
$$\pi = \frac{u^n}{\beta}$$

The resulting loss function is then:

$$Loss = u^2 + \pi^2 = (u^n)^2 + \left(\frac{u^n}{\beta}\right)^2$$